Stress-strain Relationships

Plane Stress

$$\sigma_x = \frac{E}{1-\upsilon^2}(\varepsilon_x + \upsilon\varepsilon_y)$$

$$\sigma_y = \frac{E}{1-\upsilon^2}(\varepsilon_y + \upsilon\varepsilon_x)$$

$$\tau_{xy} = G\gamma_{xy} \qquad E = 2(1+\upsilon)G$$

$$\varepsilon_x = \frac{\sigma_x - \upsilon\sigma_y}{E}$$

$$\varepsilon_y = \frac{\sigma_y - \upsilon\sigma_x}{E}$$

$$\varepsilon_z = \frac{-\upsilon}{1-\upsilon}\left(\varepsilon_x + \varepsilon_y\right)$$

Uniaxial

$$\sigma_x = E\varepsilon_x$$

$$\sigma_y = \sigma_z = 0$$

$$\varepsilon_y = \varepsilon_z = -\upsilon\varepsilon_x$$

$$\delta = \varepsilon L = \left(\varepsilon_\sigma + \varepsilon_T\right)$$

$$= \frac{\sigma L}{E} + \alpha L\Delta T = \frac{PL}{AE} + \alpha L\Delta T$$

Pressure Vessels

$$\sigma_a = \frac{pr}{2t} \qquad \textit{sphere}$$

$$\sigma_h = \frac{pr}{t} = 2\sigma_a \qquad \textit{cylinder}$$

Torsion

$$\tau_c = \frac{Tc}{J} \qquad \phi = \frac{TL}{GJ} = \frac{\tau_c L}{Gc}$$

$$J = \frac{\pi d^4}{32} = \frac{\pi r^4}{2}$$

$$1hp = 33{,}000\ ft\bullet lb/\min$$

Flexure

$$\sigma = -\frac{My}{I} \qquad \tau_v = \tau_h = \frac{VQ}{It}$$

$$I = \frac{bh^3}{12}$$

$$I = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$$

$$Q = \int y\,dA = y_c A \qquad y_c = \frac{4r}{3\pi}$$

Plane Stress

$$\sigma_n = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta$$

$$= \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{nt} = -\left(\sigma_x - \sigma_y\right)\sin\theta\cos\theta + \tau_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

$$= -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{p1,p2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Plane Strain

$$\varepsilon_n = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$= \left(\frac{\varepsilon_x + \varepsilon_y}{2}\right) + \left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)\cos 2\theta + \frac{1}{2}\gamma_{xy}\sin 2\theta$$

$$\gamma_{nt} = -2\left(\varepsilon_x - \varepsilon_y\right)\sin\theta\cos\theta + \gamma_{xy}\left(\cos^2\theta - \sin^2\theta\right)$$

$$= -\left(\varepsilon_x - \varepsilon_y\right)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

$$\varepsilon_{p1,p2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

Columns

$$P_{cr} = \frac{\pi^2 EI}{L_{eff}^2} \qquad \textit{Euler}$$

$$P_{cr} = \frac{\pi^2 EA}{\left(\frac{L_{eff}}{r}\right)^2} \qquad \textit{Euler}$$

$$\textit{Pinned} - \textit{Pinned}: \quad L_{eff} = L$$

$$\textit{Fixed} - \textit{Pinned}: \quad L_{eff} = 0.7L$$

$$\textit{Fixed} - \textit{Free}: \quad L_{eff} = 2L$$

$$\textit{Fixed} - \textit{Fixed}: \quad L_{eff} = \frac{L}{2}$$